SERIES — Answers

1 **a** $= 1 + (\frac{1}{2})(-x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(-x)^3 + \ldots$

$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$

b when $x = 0.01$, $(1 - x)^{\frac{1}{2}} \approx 1 - \frac{1}{2}(0.01) - \frac{1}{8}(0.01)^2 - \frac{1}{16}(0.01)^3$

$= 1 - 0.005 - 0.000\,012\,5 - 0.000\,000\,062\,5 = 0.994\,987\,437\,5$

$(1 - 0.01)^{\frac{1}{2}} = \sqrt{0.99} = \sqrt{\frac{9\times 11}{100}} = \frac{3}{10}\sqrt{11}$

$\therefore \sqrt{11} = \frac{10}{3} \times 0.994\,987\,437\,5 = 3.316\,624\,79$ (9sf)

2 **a** $a = \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(8)^2 = -8, \quad b = \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(8)^3 = 32$

b when $x = 0.01$, $(1 + 8x)^{\frac{1}{2}} \approx 1 + 4(0.01) - 8(0.01)^2 + 32(0.01)^3$

$= 1 + 0.04 - 0.000\,8 + 0.000\,032 = 1.039\,232$

$(1 + 0.08)^{\frac{1}{2}} = \sqrt{1.08} = \sqrt{\frac{36\times 3}{100}} = \frac{3}{5}\sqrt{3}$

$\therefore \sqrt{3} = \frac{5}{3} \times 1.039\,232 = 1.732\,05$ (5dp)

3 **a** $= 9^{\frac{1}{2}}(1 - \frac{2}{3}x)^{\frac{1}{2}} = 3(1 - \frac{2}{3}x)^{\frac{1}{2}}$

$= 3[1 + (\frac{1}{2})(-\frac{2}{3}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-\frac{2}{3}x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(-\frac{2}{3}x)^3 + \ldots]$

$= 3 - x - \frac{1}{6}x^2 - \frac{1}{18}x^3 + \ldots$

b let $x = 0.05$

$\sqrt{8.7} \approx 3 - 0.05 - \frac{1}{6}(0.05)^2 - \frac{1}{18}(0.05)^3$

$= 2.949\,576$ (7sf)

4 **a** $= 1 + (\frac{1}{3})(6x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(6x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3\times 2}(6x)^3 + \ldots$

$= 1 + 2x - 4x^2 + \frac{40}{3}x^3 + \ldots$

b when $x = 0.004$, $(1 + 6x)^{\frac{1}{3}} \approx 1 + 2(0.004) - 4(0.004)^2 + \frac{40}{3}(0.004)^3$

$= 1.007\,936\,853$

$(1 + 0.024)^{\frac{1}{3}} = \sqrt[3]{1.024} = \sqrt[3]{\frac{512\times 2}{1000}} = \frac{4}{5}\sqrt[3]{2}$

$\therefore \sqrt[3]{2} = \frac{5}{4} \times 1.007\,936\,853 = 1.259\,921$ (7sf)

5 **a** $= 1 + (-3)(2x) + \frac{(-3)(-4)}{2}(2x)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(2x)^3 + \ldots$

$= 1 - 6x + 24x^2 - 80x^3 + \ldots, \ |2x| < 1 \ \therefore$ valid for $|x| < \frac{1}{2}$

b $= (1 + 3x)(1 + 2x)^{-3} = (1 + 3x)(1 - 6x + 24x^2 - 80x^3 + \ldots)$

$= 1 - 6x + 24x^2 - 80x^3 + 3x - 18x^2 + 72x^3 + \ldots$

$= 1 - 3x + 6x^2 - 8x^3 + \ldots$

6 $\frac{2+x}{\sqrt{4-2x}} = (2 + x)(4 - 2x)^{-\frac{1}{2}} = (2 + x) \times 4^{-\frac{1}{2}}(1 - \frac{1}{2}x)^{-\frac{1}{2}}$

$= (2 + x) \times \frac{1}{2}[1 + (-\frac{1}{2})(-\frac{1}{2}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-\frac{1}{2}x)^2 + \ldots]$

$= (2 + x)(\frac{1}{2} + \frac{1}{8}x + \frac{3}{64}x^2 + \ldots)$

$\therefore$ coeff of $x^2 = (2 \times \frac{3}{64}) + (1 \times \frac{1}{8}) = \frac{7}{32}$

7 **a** $\frac{2-11x}{1-5x+4x^2} \equiv \frac{A}{1-x} + \frac{B}{1-4x}$

$2 - 11x \equiv A(1 - 4x) + B(1 - x)$

$x = 1 \quad \Rightarrow \quad -9 = -3A \quad \Rightarrow \quad A = 3$

$x = \frac{1}{4} \quad \Rightarrow \quad -\frac{3}{4} = \frac{3}{4}B \quad \Rightarrow \quad B = -1$

b $\frac{2-11x}{1-5x+4x^2} \equiv \frac{3}{1-x} - \frac{1}{1-4x}$

$\frac{3}{1-x} = 3(1 - x)^{-1} = 3[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-x)^3 + \ldots]$

$= 3 + 3x + 3x^2 + 3x^3 + \ldots, \ |-x| < 1 \ \therefore |x| < 1$

$\frac{1}{1-4x} = (1 - 4x)^{-1} = 1 + (-1)(-4x) + \frac{(-1)(-2)}{2}(-4x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-4x)^3 + \ldots$

$= 1 + 4x + 16x^2 + 64x^3 + \ldots, \ |-4x| < 1 \ \therefore |x| < \frac{1}{4}$

$\therefore \ \frac{2-11x}{1-5x+4x^2} = (3 + 3x + 3x^2 + 3x^3 + \ldots) - (1 + 4x + 16x^2 + 64x^3 + \ldots)$

$= 2 - x - 13x^2 - 61x^3 + \ldots,$ valid for $|x| < \frac{1}{4}$

8 **a** $\frac{4-17x}{(1+2x)(1-3x)^2} \equiv \frac{A}{1+2x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}$

$4 - 17x \equiv A(1 - 3x)^2 + B(1 + 2x)(1 - 3x) + C(1 + 2x)$

$x = -\frac{1}{2} \quad \Rightarrow \quad \frac{25}{2} = \frac{25}{4}A \quad \Rightarrow \quad A = 2$

$x = \frac{1}{3} \quad \Rightarrow \quad -\frac{5}{3} = \frac{5}{3}C \quad \Rightarrow \quad C = -1$

coeffs of $x^2 \ \Rightarrow \quad 0 = 9A - 6B \quad \Rightarrow \quad B = 3$

$\therefore \ f(x) \equiv \frac{2}{1+2x} + \frac{3}{1-3x} - \frac{1}{(1-3x)^2}$

b $\frac{2}{1+2x} = 2(1 + 2x)^{-1} = 2[1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(2x)^3 + \ldots]$

$= 2 - 4x + 8x^2 - 16x^3 + \ldots$

$\frac{3}{1-3x} = 3(1 - 3x)^{-1} = 3[1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-3x)^3 + \ldots]$

$= 3 + 9x + 27x^2 + 81x^3 + \ldots$

$\frac{1}{(1-3x)^2} = (1 - 3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(-3x)^3 + \ldots$

$= 1 + 6x + 27x^2 + 108x^3 + \ldots$

$f(x) = (2 - 4x + 8x^2 - 16x^3 + \ldots) + (3 + 9x + 27x^2 + 81x^3 + \ldots) - (1 + 6x + 27x^2 + 108x^3 + \ldots)$

$= 4 - x + 8x^2 - 43x^3 + \ldots$

9 **a** $(1 + ax)^b = 1 + b(ax) + \frac{b(b-1)}{2}(ax)^2 + \ldots$

$\therefore \quad ab = -6 \qquad (1)$

and $\quad \frac{1}{2}a^2b(b - 1) = 24 \quad (2)$

(1) $\Rightarrow \quad a = -\frac{6}{b}$

sub. (2) $\Rightarrow \quad \frac{18}{b}(b - 1) = 24$

$18b - 18 = 24b$

$b = -3$

$a = 2$

b $= \frac{(-3)(-4)(-5)}{3\times2}(2)^3 = -80$